SQA 2024

Paper 2

Higher Maths

Maths.scot

1 (a) Midpoint of AC $= \left(\frac{-3+11}{2}, \frac{8+0}{2}\right) = (4,4)$

$$m_{MB} = \frac{-6-4}{-1-4} = \frac{-10}{-5} = 2$$

$$y-4 = 2(x-4)$$
$$= 2x-8$$
$$y = 2x-4$$

(b) $$m_{BC} = \frac{0-(-6)}{11-(-1)} = \frac{6}{12} = \frac{1}{2}$$

$$m_{\perp} = -2$$

$$y-0 = -2(x-11)$$
$$y = -2x+22$$

(c) Point of intersection: $2x-4 = -2x+22$

$$4x = 26$$
$$x = \frac{26}{4} = \frac{13}{2}$$

When $x = \frac{13}{2}$, $y = 2\left(\frac{13}{2}\right)-4 = 13-4 = 9$ $\left(\frac{13}{2}, 9\right)$

2 $$y = \frac{8}{x^3} = 8x^{-3}$$

$$\frac{dy}{dx} = -24x^{-4} = -\frac{24}{x^4}$$

When $x=2$, $m = -\frac{24}{2^4} = -\frac{24}{16} = -\frac{3}{2}$

When $x=2$, $y = \frac{8}{2^3} = \frac{8}{8} = 1$

$$y-1 = -\frac{3}{2}(x-2)$$
$$2(y-1) = -3(x-2)$$
$$2y-2 = -3x+6$$
$$2y = -3x+8$$ (or equivalent)

③ (a) $\overrightarrow{ED} = \overrightarrow{OD} - \overrightarrow{OE} = \begin{pmatrix}2\\-3\\4\end{pmatrix} - \begin{pmatrix}1\\1\\-2\end{pmatrix} = \begin{pmatrix}1\\-4\\6\end{pmatrix}$

$\overrightarrow{EF} = \overrightarrow{OF} - \overrightarrow{OE} = \begin{pmatrix}3\\2\\1\end{pmatrix} - \begin{pmatrix}1\\1\\-2\end{pmatrix} = \begin{pmatrix}2\\1\\3\end{pmatrix}$

(b) (i) $\overrightarrow{ED}.\overrightarrow{EF} = \begin{pmatrix}1\\-4\\6\end{pmatrix}\cdot\begin{pmatrix}2\\1\\3\end{pmatrix} = 2+(-4)+18 = 16$

(ii) $|\overrightarrow{ED}| = \sqrt{1^2+(-4)^2+6^2} = \sqrt{1+16+36} = \sqrt{53}$

$|\overrightarrow{EF}| = \sqrt{2^2+1^2+3^2} = \sqrt{4+1+9} = \sqrt{14}$

$$\overrightarrow{ED}.\overrightarrow{EF} = |\overrightarrow{ED}||\overrightarrow{EF}|\cos\theta$$

$$16 = \sqrt{53}\sqrt{14}\cos\theta$$

$$\cos\theta = \frac{16}{\sqrt{53}\sqrt{14}}$$

$$\theta = \cos^{-1}\left(\frac{16}{\sqrt{53}\sqrt{14}}\right)$$

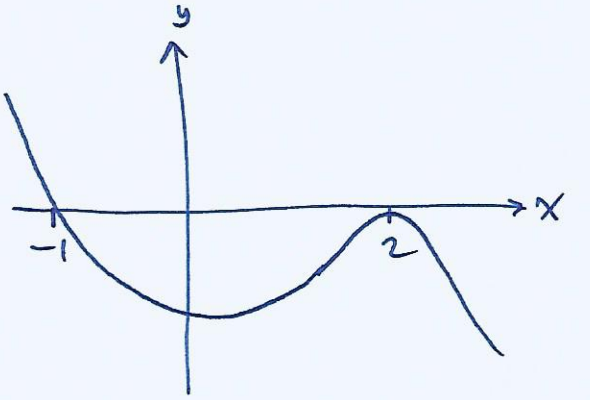
$$= 54.0288\ldots$$

$$\approx 54.0^\circ$$

④ (a) Right 4, up 2

$\therefore (-1+4, 3+2) = (3,5)$

(b)



5

$$\int_0^{\pi/7} \sin 5x \, dx = \left[-\frac{1}{5}\cos 5x \right]_0^{\pi/7}$$

$$= \left(-\frac{1}{5}\cos\frac{5\pi}{7}\right) - \left(-\frac{1}{5}\cos 0\right)$$

$$= 0.12469\ldots + \frac{1}{5}\times 1$$

$$= 0.32469\ldots$$

$$\approx 0.325$$

6

$$y = ax^b$$

$$\log_5 y = \log_5 ax^b$$

$$\log_5 y = \log_5 a + \log_5 x^b$$

$$\log_5 y = \underbrace{\log_5 a}_{c} + \underbrace{b}_{m}\log_5 x$$

$$m = \frac{10-(-2)}{4-0} = \frac{12}{4} = 3$$

$c = -2$ from the graph.

So $b = 3$

and $-2 = \log_5 a$

$$5^{-2} = a$$

$$a = \frac{1}{5^2} = \frac{1}{25}$$

7

$$\text{"Upper} - \text{lower"} = (6+4x-2x^2)-(x^3-6x^2+11x)$$

$$= 6+4x-2x^2-x^3+6x^2-11x$$

$$= -x^3+4x^2-7x+6$$

$$\text{Area} = \int_0^2 (-x^3+4x^2-7x+6)\,dx$$

$$= \left[-\frac{x^4}{4}+\frac{4x^3}{3}-\frac{7x^2}{2}+6x\right]_0^2$$

$$= \left(-\frac{2^4}{4}+\frac{4(2^3)}{3}-\frac{7(2^2)}{2}+6(2)\right)-(0)$$

$$= -\frac{16}{4}+\frac{32}{3}-\frac{28}{2}+12$$

$$= -4+\frac{32}{3}-14+12$$

$= \frac{14}{3}$ square units

Higher Maths

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8 (a) $f(g(x)) = f(x+1)$
$= 2(x+1)^2 - 18$

(b) $\frac{1}{f(g(x))}$ is undefined when $f(g(x)) = 0$
i.e. when $(x+1)^2 = 9$
$x+1 = \pm\sqrt{9}$
$x+1 = -3$ or $x+1 = 3$
$x = -4$ or $x = 2$

9 (a) $y = \frac{1}{3}x^3 - x^2 - 3x + 1$

$\frac{dy}{dx} = x^2 - 2x - 3$

SPs when $\frac{dy}{dx} = 0$: $x^2 - 2x - 3 = 0$
$(x+1)(x-3) = 0$
$x+1=0$ or $x-3=0$
$x=-1$ or $x=3$

When $x=-1$, $y = \frac{1}{3}(-1)^3 - (-1)^2 - 3(-1) + 1$
$= -\frac{1}{3} - 1 + 3 + 1$
$= \frac{8}{3}$ $\quad\left(-1, \frac{8}{3}\right)$

When $x=3$, $y = \frac{1}{3}(3^3) - 3^2 - 3(3) + 1$
$= 9 - 9 - 9 + 1$
$= -8$ $\quad(3, -8)$

(b) When $x=6$, $y = \frac{1}{3}(6^3) - 6^2 - 3(6) + 1$
$= 72 - 36 - 18 + 1$
$= 19$

x	-1	3	6
y	$\frac{8}{3}$	-8	19

Greatest value = 19

Least value = −8

(10) (a) $2g = 18$ $\quad 2f = -2$
$g = 9$ $\quad f = -1$ $\quad$ Centre $(-9, 1)$

$$r = \sqrt{g^2 + f^2 - c}$$
$$= \sqrt{9^2 + (-1)^2 - (-8)}$$
$$= \sqrt{81 + 1 + 8}$$
$$= \sqrt{90}$$
$$= \sqrt{9 \times 10}$$
$$= 3\sqrt{10}$$

(b) Let d = distance between centres.

$$d = \sqrt{(-9-(-6))^2 + (1-0)^2}$$
$$= \sqrt{3^2 + 1^2}$$
$$= \sqrt{10}$$

$$d = r_1 - r_2$$
$$\sqrt{10} = 3\sqrt{10} - r_2$$
$$\therefore\ r_2 = 2\sqrt{10}$$

$$(x+6)^2 + (y-0)^2 = (2\sqrt{10})^2$$
$$(x+6)^2 + y^2 = 40$$

(11) (a) When $t = 0$, $N = 6.8e^0 = 6.8$
Number of vehicles = 6.8 million (or 6 800 000)

(b)
$$125 = 6.8e^{10k}$$
$$e^{10k} = \frac{125}{6.8}$$
$$10k = \ln\left(\frac{125}{6.8}\right)$$
$$k = \frac{1}{10}\ln\left(\frac{125}{6.8}\right)$$
$$= 0.29113\ldots \approx 0.291$$

(12)

$$2\sin 2x - \sin^2 x = 0$$
$$2(2\sin x\cos x) - \sin^2 x = 0$$
$$4\sin x\cos x - \sin^2 x = 0$$
$$(\sin x)(4\cos x - \sin x) = 0$$

$\sin x = 0$ or $4\cos x - \sin x = 0$

$\underline{x = 0^\circ, 180^\circ}$ or $\sin x = 4\cos x$

$$\frac{\sin x}{\cos x} = 4$$
$$\tan x = 4$$

Related acute angle $= \tan^{-1} 4$
$= 75.963\ldots$
$\approx 76.0^\circ$

S	A ✓
T ✓	C

$$x = 76.0^\circ, 180 + 76.0^\circ$$
$$\underline{x = 76.0^\circ, 256.0^\circ}$$

(13)

$$f(x) = k(x+a)^2(x+b)(x+c)$$
$$f(x) = k(x-3)^2(x+1)(x-5)$$

When $x = 0$, $y = -9$

$$\therefore \quad -9 = k(-3)^2(1)(-5)$$
$$-9 = k(9)(-5)$$
$$-9 = -45k$$
$$k = \frac{-9}{-45} = \frac{1}{5}$$

$$f(x) = \frac{1}{5}(x-3)^2(x+1)(x-5)$$